Reliability Engineering – Part 13

Reliability Modeling and Prediction (B)

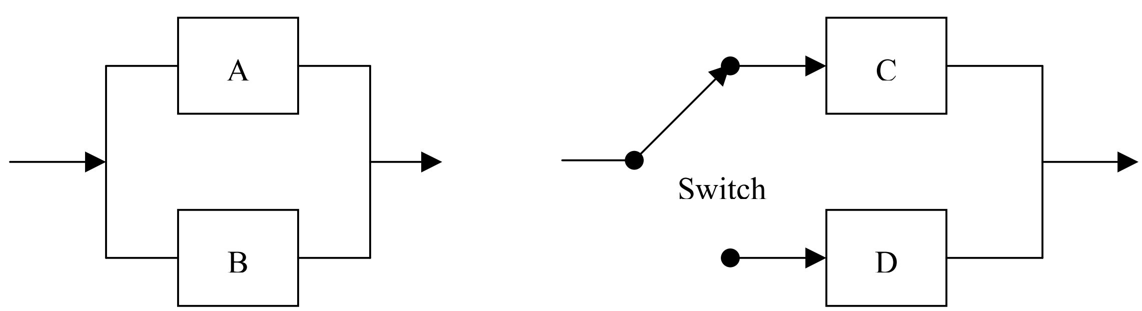
The previous section on modeling and prediction described reliability modeling of basic series and parallel systems and some more complex system structures. In this section we will review component/system redundancies in general.

Active and Passive Redundancy

Redundancy is defined as the use of additional components or sub-systems beyond the number actually required for the system to operate reliably. A basic parallel system has inherent redundancy since the failure of one or more components does not result in a system failure as long as one component remains functional.

Redundancies can be categorized as **active** or **passive** (**standby**) redundancy. In systems with **active** redundancy all redundant components are in operation and are sharing the load with the main component. Upon failure of one component, the surviving components carry the load, and as a result, the failure rate of the surviving components may be increased. See fig 13-1 (a)

The redundant or back-up components in **passive** or **standby** systems start operating only when one or more fail. The back-up components remain dormant until needed. See fig 13-1 (b)



(a) Fig 13-1 (b)

Assuming $R_{(A)} = R_{(B)} = R_{(C)} = R_{(D)}$ and a perfect switch in block diagram (b) it seems that active and passive redundant systems would have the same reliability. However, whereas components A, B, and C start operating at time t = 0, component D does not start to operate until component C fails. Hence, all things equal, the passive redundant system depicted in (b) should have a higher reliability.

Standby redundant systems can be broken down into two categories:

A) Hot standby
The standby components have the same failure rate as the primary component. The failure rate of one component is not affected by the performance/non-performance of the other components. Hence, the components are statistically independent.

B) Cold standby
The standby components have a zero failure rate. They do not fail when they are in standby mode. If and when the primary component fails, a standby component becomes the primary component with a non-zero failure rate.

Active or Shared Load Parallel Systems

The reliability of an **active, shared load**, parallel system can be calculated as follows:

$$\mathbf{R}_{(\mathbf{t})} = \mathbf{e}^{-2\lambda_1 \mathbf{t}} + \left(\frac{2\lambda_1}{2\lambda_1 - \lambda_2} \right) \left(\mathbf{e}^{-\lambda_2 \mathbf{t}} - \mathbf{e}^{-2\lambda_1 \mathbf{t}} \right)$$

where: λ_1 is the failure rate for each unit when both are working
λ_2 is the failure rate of the surviving unit when the other one has failed.

When $2\lambda_1 = \lambda_2$ the equation is reduced to $\mathbf{R}_{(\mathbf{t})} = \mathbf{e}^{-2\lambda_1 \mathbf{t}} + (2\lambda_1 \mathbf{t})(\mathbf{e}^{-\lambda_2 \mathbf{t}})$

Note: Under the exponential assumption the failure rates are constant, independent of time.

k-out-of-n Systems

In **k-out-of-n** active parallel systems any combination of (**k**) functional components out of (**n**) independent components will assure that the system functions as intended.

Most **k-out-of-n** systems have independent and identical components, which makes it relatively easy to estimate the system's reliability using the binomial distribution.

$$\mathbf{R}_{\text{system}} = \sum_{i=k}^{n} \binom{n}{i} \mathbf{R}^{i} (1 - \mathbf{R})^{n-i}$$

However, if the components are not identical, every successful path of the reliability structure must be investigated in order to accurately estimate the system's reliability.

Passive or Standby Parallel Systems

The reliability of a **standby** parallel system consisting of (**n**) identical, independent components with (**n-1**) components in stand-by mode is defined by:

$$\mathbf{R}_{(t)} = \mathbf{e}^{-\lambda t} \sum_{i=0}^{n-1} \frac{(\lambda t)^i}{i!}$$

(Assumes a perfect switch and constant failure rate).

For two identical components (primary and back-up) the formula is simplified to:

$$R_{(t)} = e^{-\lambda t}\,(1+\lambda t)$$

($R_{switch} = 1$, Exponential distribution of time to failure).

If the reliability of the switch is less than one, the reliability of the system is affected by the switching mechanism and is reduced accordingly.

$$\mathbf{R}_{(t)} = \mathbf{e}^{-\lambda t}(1 + \mathbf{R}_{sw}\lambda t)$$

(Two identical components. Exponential distribution. $R_{sw}<1$).

The reliability of a standby system consisting of one primary component with constant failure rate λ_1 and a backup component with constant failure rate λ_2 is given by:

$$\mathbf{R}_{(t)} = \mathbf{e}^{-\lambda_1 t} + \mathbf{R}_{sw}\left(\frac{\lambda_1}{\lambda_2 - \lambda_1}\right)\left(\mathbf{e}^{-\lambda_1 t} - \mathbf{e}^{-\lambda_2 t}\right)$$

($R_{sw}<1$).

Example 13.1

Four identical and independent engines, working in tandem, pull a freight train. The time to failure of the locomotive engines is Weibull distributed with $\beta = 0.8$ and $\eta = 60{,}000$ hours. At least three engines must be working for the train to be pulled at full power. What is the probability that the train will run at full capacity for at least 15,000 hours without failure?

The train's engine system represents a **k-out-of-n** active parallel system in which n= 4 and k=3.

$$\mathbf{R}_{system} = \sum_{i=k}^{n} \binom{n}{i} (\mathbf{R}_{eng})^{i} (1-\mathbf{R}_{eng})^{n-i} \qquad \mathbf{R}_{eng} = \mathbf{e}^{-\left(\frac{t}{\eta}\right)^{\beta}} = \mathbf{e}^{-\left\{\frac{15(10)^3}{6(10)^4}\right\}^{0.8}} = \mathbf{e}^{-(0.25)^{0.8}} = 0.719$$

$$\mathbf{R}_{\text{system}} = \sum_{\mathbf{i}=3}^{\mathbf{n}} \binom{4}{\mathbf{i}} (0.719)^{\mathbf{i}} (1-0.719)^{4-\mathbf{i}} = \binom{4}{3} (0.719)^3 (0.281)^{4-3} + \binom{4}{4} (0.719)^4 (0.281)^{4-4}$$

$$\mathbf{R}_{\text{system}} = 4(0.3717)(0.281) + 0.2672 = 0.41779 + 0.2672 = 0.685$$

<u>Example 13.2</u>

Two feed pumps in a nuclear power plant are connected in a stand-by mode. One is active and one is on standby. The power plant will have to shut down if both feed pumps fail. If the time to failure of each pump has an exponential distribution with $\boldsymbol{\theta} = 28{,}000$ hours, and the failure rate of the switching mechanism λ_{SW} is $(10)^{-6}$ what is the probability that the power plant will <u>not</u> have to shut down due to a pump failure in 10,000 hours?

The reliability of this standby parallel system is:

$$\mathbf{R}_{(\mathbf{t})} = \mathbf{e}^{-\lambda \mathbf{t}} (1 + \mathbf{R}_{\text{sw}} \lambda \mathbf{t}) \qquad \text{In which:} \quad \mathbf{R}_{\text{sw}} = \mathbf{e}^{-(10)^4 (10)^{-6}} = 0.99$$

$$\mathbf{R}_{(\text{System})} = \mathbf{e}^{-\left(\frac{10^4}{2.8(10)^4}\right)} \left[1 + (0.99) \left(\frac{(10)^4}{2.8(10)^4} \right) \right] = 0.69967\,(1 + 0.35357) = 0.947$$

Note: Few mechanical components or systems have a constant failure rate. A more likely time to failure distribution for electro-mechanical devices would be Lognormal or Weibull. Calculating the reliability of standby systems of which the components have a Lognormal or Weibull time-to-failure distribution is more complex and beyond the scope of this text.

References:

- Reliability Engineering – E.A. Elsayed
- Practical Reliability Engineering – Patrick O'Connor
- Reliability Toolkit – Reliability Analysis Center
- The CRE Primer – Robert Dovich and Bill Wortman